EXACT EQUATIONS AND TRANSPORT COEFFICIENTS FOR A MULTICOMPONENT GAS MIXTURE WITH A PARTIALLY IONIZED PLASMA

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Molecular transport processes are governing in many problems associated with the flow of high-temperature multicomponent gas mixtures. Thus, a flow with a partially dissociated and ionized multicomponent gas is realized in problem of viscous heat-conducting gas flow around bodies in practically the whole flow domain outside the bow shock on the most heat-stressed section of the trajectory, where convective heat transfer predominates, during entry in the atmosphere of planets of the solar system. Molecular transport processes in such a mixture govern directly the molecular heat and mass fluxes, the viscous friction force, as well as the rate of thermochemical destruction of the streamlined surface [1]. Transport processes play an important role in the flow of a weakly ionized plasma in a MHD generator [2], gas-phase nuclear reactors [3], and other problems. Consequently, a sufficiently rigorous and complete study of the behavior of the transfer coefficients in a broad temperature and pressure range is a problem of practical importance, and even more so since experimental investigations of multicomponent plasmas at high temperatures are related to great complexities in the diagnostics. It is also important to note that a study of all the necessary properties of a complete set of transfer coefficients has not been performed up to now because of the complexity of the appropriate formulas given by the kinetic theory of gases in the exposition in [4]. Of the experimental papers, [5-7] should be noted, in which the investigation of certain transfer properties of air, nitrogen, hydrogen, and argon was performed (mainly the heat conduction and viscosity coefficients only). Numerical computations were carried out in [8-11] for the transport properties of ionized gases by kinetic theory methods [4] with high approximations taken into account in a polynomial expansion of the distribution functions in Sonin polynomials. However, for flows of chemically equilibrium high-temperature gases for which effective coefficients are introduced in the hydrodynamics equations with additional terms associated with component transfer, heat liberation in reactions, and the use of quasineutrality conditions of the ionized mixtures taken into account (a strict determination of the complete set of effective transfer coefficients is given in [12, 13]), computations of all the effective coefficients in the hydrodynamics equations have never been performed successively. Up to now there is not sufficient information in the literature about the influence of any coefficient in any element temperature, pressure, and concentration range, nor about the role of taking account of high approximation in the computation of the transport coefficients for determining the heat flux, the component diffusion fluxes, and the viscous friction force [14].

The molecular-kinetic approach to describing transport processes in gas mixtures, which is based on solving the system of Boltzmann equations for the component distribution functions by the Chapman–Enskog method, permits the derivation of the complete set of hydrodynamics equations and obtaining expressions for the known particle interaction potentials for all coefficients in these equations [4]. The problem of calculating the transfer coefficients for such an approach is separated from the solution of the fundamental hydrodynamic problem: The transport coefficients are functions of just the temperature, pressure, and chemical composition. The known procedure for expanding the distribution function perturbations in Sonin polynomials results in an infinite system of algebraic equations, for which the results of its solution depend on the number of terms ξ in the expansion. The number of terms ξ for which the necessary accuracy of the transport coefficients is assumed depends on the number of conditions and, in particular, is determined by the nature of the behavior of the component interaction section. For neutral gas mixtures the first nonzero approximation turns out to be completely adequate. For ionized gases and gas mixtures containing a volatile component, the number of terms ξ assuring convergence will grow: Thus, utilization of the least nonzero approximation in computing the heat conduction coefficient of ionized air with a molar elec-

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tron concentration of $r_E \approx 0.5$ results in a $\approx 60\%$ error, and in an error of $\approx 11\%$ [15] in the computation of the heat conduction coefficient of a mixture containing a volatile component at T = 200°K. For a fully ionized gas the number of approximations ξ assuring the value of the transport coefficients with 1% accuracy and agreement with the asymptotic values obtained in solving the Fokker–Planck equations for an ideal plasma [16] is four.

The expressions for the transport coefficients have the form of a ratio of determinants of the order $N\xi + 1$ and $N\xi$, where N is the number of mixture components in the traditional approach to the solution of algebraic equations obtained in the Chapman–Enskog method by using series expansions in Sonin polynomials. The order of the determinant is raised by N when the order of the approximation ξ is increased by one. This produces substantial numerical difficulties in computing the transport properties of multicomponent mixtures in high approximations. In a different approach to the solution of the initial algebraic system of equations to determine the coefficients of the Sonin polynomial expansions, proposed in [17, 18], the mass transport equation of the components and the energy transport equation are written in a form solved for the "force" vectors in terms of the diffusion and heat fluxes. In such a representation the expressions for the transport coefficients in high approximations have a number of advantages over the expressions written in the traditional form: For a given approximation ξ the order of the determinants in the transport coefficients is reduced by a number equal to the number of mixture components; the procedure of double inversion of the matrices in any approximation, which takes place in the traditional calculation of the true heat conduction coefficient of the mixture, is eliminated completely [4]; substantially simpler resistance coefficients $\Delta_{ij}(\xi)$, for the calculation, written in terms of the binary diffusion coefficients $\mathscr{D}_{ij}(1)$ in a first approximation and correction factors $\varphi_{ij}(\xi)$ dependent on the high approximations (because of writing the mass transport equations of the components in the form of Stefan–Maxwell relations in any approximation ξ) are introduced in place of the multicomponent diffusion coefficients $D_{ij}(\xi)$; simpler thermodiffusion ratios $k_{Ti}(\xi)$ for the calculations are introduced in place of the multicomponent thermodiffusion coefficients D_i^T. A modified form of writing the transport equations is obtained for arbitrary flows of multicomponent mixtures in the most simple form, which permits execution of extensive computations of the transport coefficients in this paper in a broad range of pressures, temperatures, and concentrations of the elements. The new form of the transfer equations is quite convenient for the numerical solution of diverse hydrodynamic problems [19, 20]. The structure used in this paper for the transport equations is moreover obtained phenomenologically also, by using methods of the thermodynamics of irreversible processes [21, 22].

1. SYSTEM OF NAVIER–STOKES EQUATIONS FOR MULTICOMPONENT GAS MIXTURES

Let us consider an N-component gas mixture. For the practical computation of flows with chemical reactions proceeding arbitrarily, it is convenient to extract L independent (basis) components, for which, for instance, the chemical elements and an electronic component can be taken, and R = N − L components (reaction products). Let B_j (j = 1, ..., N) be the chemical symbol of the basis component, and A_i (i = L+1, ..., N), the symbol of the reaction product. Then without limiting the generality the linearly independent stoichiometric equations of the reaction can be written in the form

$$A_i = \sum_{j=1}^{L} \nu_{ij} B_j - q_i(T), \quad i = L+1, \ldots, N, \tag{1.1}$$

where ν_{ij} are the stoichiometric reaction coefficients, and q_i are the heats of reaction. In conformity with (1.1), the mass and charge conservation laws in the reactions will have the form

$$m_i = \sum_{j=1}^{L} \nu_{ij} m_j, \quad e_i = \sum_{j=1}^{L} \nu_{ij} e_j, \tag{1.2}$$

where m_i is the mass of the i-th particle, and e_i is the charge of the i-th particle. We determine the element concentrations c_j^* (j = 1, ..., L) and the element diffusion fluxes J_j^* (j = 1, ..., L) in conformity with the stoichiometric mode of writing the reaction (1.1) as follows

$$c_j^* = c_j + \sum_{k=L+1}^{N} \nu_{kj} \frac{m_j}{m_k} c_k, \quad \mathbf{J}_j^* = \mathbf{J}_j + \sum_{k=L+1}^{N} \nu_{kj} \frac{m_j}{m_k} \mathbf{J}_k, \quad \sum_{j=1}^{L} c_j^* = 1, \tag{1.3}$$
$$\sum_{j=1}^{L} \mathbf{J}_j^* = 0.$$

Then the system of Navier–Stokes equations can be written in the form [12]

$$\frac{\partial\rho}{\partial t} + \operatorname{div}(\rho\mathbf{v}) = 0, \quad \rho\mathbf{v} = \sum_{k=1}^{N} \rho_k\mathbf{v}_k; \tag{1.4}$$

$$\rho\frac{dc_j^*}{dt} + \operatorname{div}\mathbf{J}_j^* = 0, \quad \rho = \sum_{k=1}^{N} \rho_k \quad (j = 1, \ldots, L); \tag{1.5}$$

$$\rho\frac{dc_i}{dt} + \operatorname{div}\mathbf{J}_i = \dot{w}_i \quad (i = L+1, \ldots, N); \tag{1.6}$$

$$\rho\frac{d\mathbf{v}}{dt} = \sum_{k=1}^{N} \rho_k\mathbf{F}_k - \nabla p + \operatorname{div}\hat{\tau}; \tag{1.7}$$

$$\rho\frac{d}{dt}\left(h + \frac{v^2}{2}\right) = \frac{\partial p}{\partial t} + \sum_{k=1}^{N} \rho_k v_k\mathbf{F}_k + \operatorname{div}(\hat{\tau}\cdot\mathrm{v} - \mathbf{J}_q); \tag{1.8}$$

$$p = \frac{\rho}{m}kT, \quad \frac{1}{m} = \sum_{k=1}^{N}\frac{c_k}{m_k}, \quad \rho = \sum_{k=1}^{N} n_k m_k, \quad n = \sum_{k=1}^{N} n_k, \tag{1.9}$$

where $\rho_i = n_i m_i$, n_i, $\mathbf{v}_i$, $c_i = \rho_i/\rho$, $x_i = n_i/n$, J_i, w_i, $\mathbf{F}_i$, h_i are the mass density, the number of particles per unit volume, the mean statistical veocity, the mass concentration, the molar concentration, the mass diffusion flux vector of the i-th components, the rate of mass origination for the i-th component per unit volume per unit time, the mass force acting on the i-th component, the specific enthalpy of the i-th component, respectively; ρ, $\mathbf{v}$, p, h, T, m, n, J_q, $\hat{\tau}$ are the density, mean-statistical velocity, pressure, enthalpy, temperature, mass, the total number of particles per unit volume, the total heat flux vector, the viscous stress tensor of the mixture as a whole, respectively, and k is the Boltzmann constant. Equation (1.4) is the continuity equation for the mixture as a whole, (1.5) is the diffusion equation of the elements, (1.6) is the diffusion equation of the component-reaction products, (1.7) is the momentum equation for the mixture as a whole, (1.8) is the energy equation for the mixture as a whole, and (1.9) is the equation of state for a mixture of ideal (not dense) gases. By virtue of the charge conservation law (1.2) in the chemical reactions (1.1), we have

$$e_i = \nu_{iE}e_E, \quad \rho_e = \sum_{k=1}^{N} n_k e_k = \frac{\rho e_E}{m_E}c_E^*; \quad c_E^* = \frac{m_E}{m}x_E^*,$$

where the subscript E denotes electron, and ρ_e is the volume charge density.

To close the system of equations (1.4)-(1.9), it is necessary to have explicit expressions for the "fluxes" J_i ($i = 1, \ldots, N$), J_q, τ, $\dot{w}_i$, the so-called transport equations. Phenomenological and kinetic [21, 22] approaches yield expressions of perfectly identical structure for the transport equations.

The equation for the viscous stress tensor ("momentum transport") is

$$\tau_{\alpha\beta} = -\frac{2}{3}\mu(\xi)\operatorname{div} v\delta_{\alpha\beta} + 2\mu(\xi)e_{\alpha\beta}, \quad e_{\alpha\beta} = \frac{1}{2}\left(\frac{\partial v_\alpha}{\partial x_\beta} + \frac{\partial v_\beta}{\partial x_\alpha}\right), \tag{1.10}$$

where $\tau_{\alpha\beta}$ are the viscous stress tensor components, $e_{\alpha\beta}$ are the strain rate tensor components, and $\mu(\xi)$ is the mixture viscosity coefficient in the approximation ξ. The presence of chemical reactions causes the appearance of an additional member in (1.10) with the volume viscosity coefficient. However, for aerodynamic problems this member is negligible [23]. The mass transport equations of the separate components are written according to [17], in a form that is solved at once for the "forces" in terms of the fluxes in any approximation ξ. Following the writing of (1.5) and (1.6), we represent the equations for the diffusion fluxes of the components separately for the elements J_j^* ($j = 1, \ldots, L$) and separately for the reaction products J_i ($i = L+1, \ldots, N$) by eliminating the diffusion fluxes of the component-elements J_j ($j = 1, \ldots, L$) by using (1.3):

$$\mathbf{d}_j = -\Delta_j\mathbf{J}_j^{*\prime} + x_j\sum_{l=1}^{L}\Delta_{jl}\mathbf{J}_l^{*\prime} + \sum_{k=L+1}^{N}\theta_{kj}^*\mathbf{J}_k^{\prime}, \quad \mathbf{J}_k^{\prime} = \frac{1}{m}\mathbf{J}_k \tag{1.11}$$
$$(j = 1, \ldots, L);$$

$$\mathbf{d}_i = -\Delta_i \mathbf{J}_i' + x_i \sum_{l=1}^{L} \Delta_{il} \mathbf{J}_l^{*\prime} + \sum_{k=L+1}^{N} \theta_{ki}^{0} \mathbf{J}_k' \qquad (i = L+1, \ldots, N), \tag{1.12}$$

where the vectors of the diffusion forces $\mathbf{d}_k$ equal

$$\mathbf{d}_k = \nabla x_k + k_{pk} \nabla \ln p + k_{\mathrm{T}k}(\xi) \nabla \ln T - \frac{c_k}{p}\left(\rho \mathbf{F}_k - \sum_{i=1}^{N} \rho_i \mathbf{F}_i\right) \quad (k = 1, \ldots, N).$$

Here

$$\Delta_{ij}^{-1} = n\mathscr{D}_{ij}(1) f_{ij}(\xi); \quad f_{ij}(\xi) = [1 + \varphi_{ij}(\xi)]^{-1}; \quad \Delta_i = \sum_{k=1}^{N} x_k \Delta_{ik};$$

$$k_{pi} = x_i - c_i; \quad \theta_{kj}^{*} = \Delta_j \nu_{kj} + x_j \left(\Delta_{jk} - \sum_{l=1}^{L} \nu_{kl} \Delta_{lj}\right);$$

$$\alpha_{\mathrm{T}i} = k_{\mathrm{T}i}/x_i; \quad \theta_{ki}^{0} = x_i \left(\Delta_{ik} - \sum_{l=1}^{L} \nu_{kl} \Delta_{li}\right);$$

Δ_{ij} are the resistance coefficients, k_{pi} are the barodiffusion coefficients, k_{Ti} are the thermodiffusion ratios, α_{Ti} is the thermodiffusion factor, $\mathscr{D}_{ij}(1)$ are the binary diffusion coefficients in a first approximation, and $\varphi_{ij}(\xi)$ are correction factors to the resistance coefficients which depend on the order of the approximation ξ [17].

The equation for the total heat flux transport has the form

$$\mathbf{J}_q = \mathbf{q} + \sum_{k=1}^{N} h_k \mathbf{J}_k = \mathbf{q} + \sum_{l=1}^{L} h_l \mathbf{J}_l^{*} - \sum_{k=L+1}^{N} q_k \mathbf{J}_k,$$

where $\mathbf{q}$ is the reduced heat flux equal to

$$\mathbf{q} = -\lambda(\xi) \nabla T + kT \sum_{k=1}^{N} \alpha_{\mathrm{T}k}(\xi) \mathbf{J}_k' = -\lambda(\xi) \nabla T + kT \sum_{l=1}^{L} \alpha_{\mathrm{T}l} \mathbf{J}_l^{*\prime} +$$

$$+ kT \sum_{k=L+1}^{N} \beta_{\mathrm{T}k}(\xi) \mathbf{J}_k', \quad \beta_{\mathrm{T}i} = \alpha_{\mathrm{T}i} - \sum_{j=1}^{L} \nu_{ij} \alpha_{\mathrm{T}j}.$$

Here $\lambda(\xi)$ is the so-called "true" heat conduction coefficient, i.e., the coefficient in front of the temperature gradient when all the diffusion fluxes vanish.

The transport coefficients in these equations have the following form:

the coefficient of "true" heat conduction

$$\begin{array}{l} \lambda(\xi) = -\dfrac{75}{8} \dfrac{nk}{\det \| \mathbf{q}_{rs} \|} \\ \quad \xi \geqslant 2 \\ \lambda(1) \equiv 0 \end{array} \begin{vmatrix} 0 & x_s & 0 & \ldots & 0 \\ x_r & \mathbf{q}_{rs}^{1,1} & \mathbf{q}_{rs}^{1,2} & \ldots & \mathbf{q}_{rs}^{1,\xi-1} \\ 0 & \mathbf{q}_{rs}^{2,1} & \mathbf{q}_{rs}^{2,2} & \ldots & \mathbf{q}_{rs}^{2,\xi-1} \\ & \ldots & & & \\ 0 & \mathbf{q}_{rs}^{\xi-1,1} & \mathbf{q}_{rs}^{\xi-1,2} & \ldots & \mathbf{q}_{rs}^{\xi-1,\xi-1} \end{vmatrix}; \tag{1.13}$$

the thermodiffusion ratios

$$\begin{array}{l} k_{\mathrm{T}i}(\xi) = -\dfrac{5}{2} \dfrac{1}{\det \| \mathbf{q}_{rs} \|} \\ \quad \xi \geqslant 2 \\ k_{\mathrm{T}i}(1) \equiv 0 \quad (i = 1, \ldots, N) \end{array} \begin{vmatrix} 0 & x_s & 0 & \ldots & 0 \\ \mathbf{q}_{ri}^{1,0} & \mathbf{q}_{rs}^{1,1} & \mathbf{q}_{rs}^{1,2} & \ldots & \mathbf{q}_{rs}^{1,\xi-1} \\ \ldots & & & & \\ \mathbf{q}_{ri}^{\xi-1,0} & \mathbf{q}_{rs}^{\xi-1,1} & \mathbf{q}_{rs}^{\xi-1,2} & \ldots & \mathbf{q}_{rs}^{\xi-1,\xi-1} \end{vmatrix}; \tag{1.14}$$

the correction factors $\varphi_{ij}(\xi)$ to the resistance coefficients

$$\begin{array}{l} \varphi_{ij}(\xi) = \dfrac{2}{3} \dfrac{\mathscr{D}_{ij}(1)}{x_i x_j} \dfrac{1}{\det \| \mathbf{q}_{rs} \|} \\ \quad \xi \geqslant 2 \\ \varphi_{ij}(1) \equiv 0 \quad (i, j = 1, \ldots, N) \end{array} \begin{vmatrix} 0 & \mathbf{q}_{js}^{0,1} & \ldots & \mathbf{q}_{js}^{0,\xi-1} \\ \mathbf{q}_{ri}^{1,0} & \mathbf{q}_{rs}^{1,1} & \ldots & \mathbf{q}_{rs}^{1,\xi-1} \\ \ldots & & & \\ \mathbf{q}_{ri}^{\xi-1,0} & \mathbf{q}_{rs}^{\xi-1,1} & \ldots & \mathbf{q}_{rs}^{\xi-1,\xi-1} \end{vmatrix}. \tag{1.15}$$

The expression for the viscosity coefficient has the form obtained in [8]

$$\mu(\xi) = -\frac{5}{2}\frac{n}{\det\|\hat{\mathbf{q}}_{rs}\|}\begin{vmatrix} 0 & x_s\sqrt{m_s} & 0 & \ldots & 0 \\ x_r\sqrt{m_r} & \hat{\mathbf{q}}_{rs}^{0,0} & \hat{\mathbf{q}}_{rs}^{0,1} & \ldots & \hat{q}_{rs}^{0,\xi-1} \\ 0 & \hat{\mathbf{q}}_{rs}^{1,0} & \hat{\mathbf{q}}_{rs}^{1,1} & \ldots & \hat{\mathbf{q}}_{rs}^{1,\xi-1} \\ & \cdot\;\cdot\;\cdot & & & \\ 0 & \hat{\mathbf{q}}_{rs}^{\xi-1,0} & \hat{\mathbf{q}}_{rs}^{\xi-1,1} & \ldots & \hat{\mathbf{q}}_{rs}^{\xi-1,\xi-1} \end{vmatrix}. \tag{1.16}$$

In these expressions the q_{rs}^{mp} and $\hat{q}_{rs}^{mp}$ are square matrices of order N × N comprised of q_{ij}^{mp} and $\hat{q}_{ij}^{mp}$, and $m, p = 0, 1, \ldots, \xi - 1$; $\det\|q_{rs}\|$ is the determinant of the matrix compiled from the matrix of the numerator by eliminating the first row and first column. The elements q_{ij}^{mp} are expressed in terms of the collision integrals $Q_{ij}^{l,s}(T)$ of different pairs of components by a known method [4, 8]. In this paper the expressions q_{ij}^{mp} differ from those presented in [8] by the factor $(m_j/2\pi kT)^{1/2}n^{-1}$ and have the form

$$q_{ij}^{mp} = \sqrt{\frac{m_j}{2\pi kT}}\frac{1}{n}\left\{8n_i\left(\frac{m_i}{m_j}\right)^{p+1/2}\sum_{k=1}^{N} n_k \sum_{l=1}^{m+1}\left[\delta_{ij} + (-1)^l\delta_{jk}\right] \times\right.$$
$$\left.\times \sum_{t=0}^{m+1-l}\frac{(m_j/m_k)^{l+2t-1}}{(m_i/m_k+1)^{m+p+1/2}}\sum_{s=l}^{m+p-(l-2)-2t} a_{lts}^{mp} Q_{ik}^{l,s}(T)\right\}, \qquad m \geqslant 0,\ p \geqslant 1.$$

The expressions for $\hat{q}_{ij}^{mp}$ also differ from those in [8] by the same factor. Values of the coefficients a_{lts}^{mp} are presented in [24].

The collision integrals $Q_{ij}^{l,s}$ of the required orders l and s are computed in the known scattering sections by the formula

$$Q_{ij}^{l,s}(T) = \frac{4(l+1)}{(s+1)!\,[2l+1-(-1)^l]}\int_0^\infty e^{-\gamma_{ij}^2}\gamma_{ij}^{2s+3}Q_{ij}^l(g)\,d\gamma_{ij},$$

where the section Q_{ij}^l can be computed if the potential of pairwise interaction of the particles (r_{ij}) is known by which the angles of particle deflection χ_{ij} [4] are computed in the classical case, and the scattering phases are computed in the quantum-mechanical case [25]. For the classical case [4]

$$Q_{ij}^l = 2\pi\int_0^\infty (1 - \cos^l\chi_{ij})\,b\,db.$$

It is seen from the expressions presented for the coefficients $\lambda(\xi)$, $k_{Ti}(\xi)$, $\varphi_{ij}(\xi)$ that the order of their determinants equals $N(\xi - 1) + 1$ and $N(\xi - 1)$ and is less by N than in the corresponding coefficients obtained by traditional means [4].

2. NUMERICAL COMPUTATION OF THE TRANSPORT COEFFICIENTS $\mu(\xi)$, $\lambda(\xi)$, $k_{Ti}(\xi)$, $\varphi_{ij}(\xi)$ OF IONIZED AIR IN HIGH APPROXIMATIONS

For the mass transport equations presented in Sec. 1 for the momentum and energy components, a method and program for computing the transport coefficients $\mu(\xi)$, $\lambda(\xi)$, $k_{Ti}(\xi)$, $\varphi_{ij}(\xi)$ of multicomponent gas mixtures were developed from (1.13)-(1.16) to a fourth approximation in ξ inclusively, in a broad temperature and pressure range. The numerical computations were executed for a chemically equilibrium partially ionized mixture of nitrogen and oxygen with the components O, N, E, O_2, N_2, NO, N^+, O^+, NO^+ taken into account for a given element composition of $c_O^* = 0.244$ and $C_N^* = 0.756$ in the $300 \leq T \leq 20{,}000$°K temperature range and $1.013 \cdot 10^2$ Pa $\leq p \leq 1.013 \cdot 10^7$ Pa pressure range. The equilibrium constants were approximated from tabulated data [26]. Double ionization and nonideal effects were not taken into account in the computation of the composition. The collision integrals $Q_{ij}^{l,s}$ of the required orders l and s were taken from [10] for all component pairs.

The identities $\sum\limits_{i=1}^{N} k_{Ti}(\xi) = 0$, $\varphi_{ij}(\xi) = \varphi_{ji}(\xi)$ were used in numerical computations of the coefficients $k_{Ti}(\xi)$ and $\varphi_{ij}(\xi)$ and the calculations were performed for those components whose concentrations and influence on the mixture properties are not negligible. The results of numerical computations of the coefficients $\mu(\xi)$, $\lambda(\xi)$, $k_{Ti}(\xi)$, and $\varphi_{ij}(\xi)$ in different approximations are represented in Tables 1-3. At low temperatures (up to the beginning of

TABLE 1

$\mu(\xi)$, mP

	$p=0,01$			$p=1,0$			$p=100,0$		
T, °K	x_E	$\xi=1$	$\xi=2$	x_E	$\xi=1$	$\xi=2$	x_E	$\xi=1$	$\xi=2$
5 000	$1,2\cdot10^{-4}$	1,588	1,591	$4,3\cdot10^{-5}$	1,571	1,574	0	1,518	1,521
6 000	$6,10\cdot10^{-4}$	1,785	1,789	$2,14\cdot10^{-4}$	1,847	1,850	$6,4\cdot10^{-5}$	1,799	1,803
7 000	$4,44\cdot10^{-3}$	1,996	2,003	$6,65\cdot10^{-4}$	2,101	2,105	$2,25\cdot10^{-4}$	2,069	2,073
8 000	0,0225	2,138	2,147	$2,39\cdot10^{-3}$	2,301	2,307	$5,6\cdot10^{-4}$	2,340	2,344
9 000	0,0772	1,951	1,967	$8,37\cdot10^{-3}$	2,508	2,516	$1,19\cdot10^{-3}$	2,608	2,612
10 000	0,189	1,356	1,372	0,0234	2,650	2,661	$2,59\cdot10^{-3}$	2,855	2,855
11 000	0,330	0,697	0,708	0,0538	2,611	2,634	$5,73\cdot10^{-3}$	3,070	3,078
12 000	0,431	0,293	0,300	0,105	2,340	2,363	0,0117	3,278	3,286
13 000	0,476	0,126	0,131	0,177	1,871	1,896	0,0215	3,509	3,464
14 000	0,491	0,0701	0,0742	0,260	1,351	1,374	0,0364	3,572	3,584
15 000	0,496	0,0533	0,0578	0,340	0,905	0,924	0,0571	3,607	3,622
16 000	0,498	0,0503	0,0552	0,402	0,585	0,066	0,0839	3,540	3,559
17 000	0,499	0,0526	0,0582	0,443	0,384	0,396	0,117	3,368	3,393
18 000	0,500	0,0574	0,0636	0,468	0,270	0,281	0,155	3,107	3,137
19 000	0,500	0,0634	0,0704	0,481	0,209	0,221	0,196	2,786	2,822
20 000	0,500	0,0702	0,0781	0,489	0,179	0,191	0,240	2,440	2,480

TABLE 2

$\lambda(\xi)$, W/(°K·m)

	$p=0,01$			$p=1$			$p=100$		
T, °K	$\xi=2$	$\xi=3$	$\xi=4$	$\xi=2$	$\xi=3$	$\xi=4$	$\xi=2$	$\xi=3$	$\xi=4$
5 000	0,252	0,253	0,253	0,221	0,221	0,221	0,187	0,188	0,188
6 000	0,379	0,381	0,387	0,287	0,287	0,287	0,240	0,241	0,241
7 000	0,479	0,501	0,514	0,402	0,403	0,404	0,298	0,298	0,298
8 000	0,542	0,606	0,616	0,545	0,551	0,563	0,376	0,376	0,376
9 000	0,520	0,630	0,635	0,677	0,717	0,735	0,490	0,489	0,489
10 000	0,424	0,578	0,580	0,789	0,891	0,908	0,652	0,654	0,654
11 000	0,352	0,551	0,551	0,863	1,045	1,058	0,864	0,870	0,880
12 000	0,341	0,587	0,587	0,888	1,161	1,169	1,106	1,134	1,158
13 000	0,372	0,668	0,669	0,890	1,259	1,263	1,363	1,449	1,484
14 000	0,423	0,774	0,774	0,905	1,372	1,375	1,631	1,812	1,854
15 000	0,484	0,894	0,894	0,950	1,518	1,520	1,904	2,218	2,263
16 000	0,551	1,026	1,026	1,025	1,694	1,696	2,179	2,660	2,705
17 000	0,624	1,168	1,168	1,122	1,895	1,896	2,453	3,132	3,175
18 000	0,703	1,322	1,322	1,236	2,115	2,116	2,729	3,633	3,674
19 000	0,787	1,487	1,487	1,362	2,352	2,353	3,012	4,163	4,202
20 000	0,877	1,662	1,622	1,498	2,604	2,605	3,307	4,723	4,759

ionization), the computed values of $\mu(\xi)$ and $\lambda(\xi)$ are in good agreement with the data in [27]. At high temperatures (in the partial-ionization domain) the computed values of $\mu(\xi)$ and $\lambda(\xi)$ are in good agreement with the results of computations [10] (for the identical air composition), which are, in turn, in good agreement with experimental data [5], and with the Spitzer theory in the domain of full ionization [16].

To investigate the convergence of values of the transport coefficients as the number of the approximation ξ increases in the whole temperature and pressure range under consideration, the computations were performed for all values of ξ = 1, 2, 3, and 4. Convergence of the coefficients $\mu(\xi)$ and $\lambda(\xi)$ for air was examined in [8, 10], while convergence of the thermodiffusion coefficient $D_i^T(\xi)$ was investigated in [8]. A detailed investigation of the convergence of the coefficients $k_{Ti}(\xi)$ and $\varphi_{ij}(\xi)$ for a multicomponent ionized mixture has not been performed earlier in general.

As is seen from Table 1, for neutral and partially ionized air ($x_E \leqslant 0.4$), the viscosity coefficient can be calculated to ≈3% error in a first approximation ($\xi = 1$). For strongly ionized air ($x_E \geqslant 0.4$), a computation of the viscosity coefficient in a first approximation results in up to 10% error; however, the second approximation already turns out to be sufficiently exact (a further increase in ξ does not affect the accuracy of computing $\mu(\xi)$ in 1% limits).

Computation of the coefficient $\lambda(\xi)$ in a second approximation for weakly ionized air yields an error of ⩽5%, while for a strongly ionized mixture ($x_E \approx 0.5$, $p = 0.01$), the

TABLE 3

		$h_{Ti}(\xi)$, $p=1$			$\varphi_{ij}(\xi)$, $p=1$					
T, °K	ξ	N_2	N	N^+	O—N	E—N	$N—O^+$	$N—N^+$	$E—N^+$	$O^+—N^+$
5 000	2	$1{,}07\cdot10^{-3}$	$-1{,}68\cdot10^{-3}$	0	—0,0019	—0,217				
	3	$1{,}08\cdot10^{-2}$	$-1{,}71\cdot10^{-3}$	0	—0,0021	—0,327				
	4	$1{,}08\cdot10^{-2}$	$-1{,}71\cdot10^{-3}$	0	—0,0021	—0,199				
8 000	2	$5{,}27\cdot10^{-3}$	$-5{,}71\cdot10^{-3}$	$-3{,}43\cdot10^{-4}$	—0,0009	0,069	—0,0116	—0,0113	—0,105	0,0361
	3	$5{,}40\cdot10^{-3}$	$-5{,}99\cdot10^{-3}$	$-2{,}12\cdot10^{-4}$	—0,0009	0,077	—0,0126	—0,0116	—0,179	0,0369
	4	$5{,}71\cdot10^{-3}$	$-6{,}67\cdot10^{-3}$	$-7{,}45\cdot10^{-5}$	—0,0010	0,106	—0,0127	—0,0117	—0,279	0,0374
13 000	2	0	$-4{,}80\cdot10^{-2}$	$-4{,}48\cdot10^{-2}$	—0,0022	0,196	—0,0608	—0,0086	—0,338	—0,0570
	3	0	$-4{,}88\cdot10^{-2}$	$-3{,}09\cdot10^{-2}$	—0,0024	0,181	—0,0624	—0,0091	—0,346	—0,0571
	4	0	$-4{,}95\cdot10^{-2}$	$-2{,}96\cdot10^{-2}$	—0,0025	0,182	—0,0626	—0,0093	—0,349	—0,0571
18 000	2	0	$-7{,}25\cdot10^{-3}$	$-2{,}48\cdot10^{-1}$	—0,0025	0,168	—0,0420	—0,0128	—0,351	—0,0742
	3	0	$-7{,}44\cdot10^{-3}$	$-2{,}19\cdot10^{-1}$	—0,0028	0,154	—0,0445	—0,0137	—0,357	—0,0752
	4	0	$-7{,}47\cdot10^{-3}$	$-2{,}18\cdot10^{-1}$	—0,0028	0,154	—0,0446	—0,0138	—0,360	—0,0752

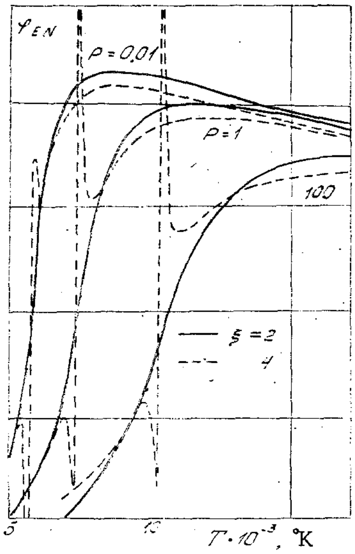

Fig. 1

value of $\lambda(2)$ in a second approximation turns out to be approximately halved as compared with the exact value. However, the third approximation already turns out to be sufficiently exact ($\lambda(3)$ agrees with $\lambda(4)$ to 1-2% accuracy in the whole range under consideration).

Convergence of the coefficients $k_{Ti}(\xi)$ and $\varphi_{ij}(\xi)$ is of a different nature for a different degree of ionization for different kinds of components. Computed values of certain of these coefficients are presented in Table 3 for p = 1 and four values of the temperature, corresponding to very weak, partial, and full ionization. As is seen from Table 3, for many components the coefficients $k_{Ti}(\xi)$ and $\varphi_{ij}(\xi)$ are negligible in the whole temperature range. Only $k_{Tc}(\xi)$, $k_{TE}(\xi)$, $\varphi_{Ec}(\xi)$, $\varphi_{En}(\xi)$ (the subscripts c, n, and E denote charge, neutral, and electron, respectively) can exert noticeable influence on the mixture properties. It should also be noted that the coefficients $\varphi_{ij}(\xi)$ enter into all the transport equations in the form of the expression $x_i[1 + \varphi_{ij}(\xi)]$; hence, to estimate the influence of the convergence of $\varphi_{ij}(\xi)$ on the mixture properties is is necessary to examine the product $x_i\varphi_{ij}(\xi)$. For partially and fully ionized air ($x_E \geqslant 0.1$), the coefficients $k_{Tc}(\xi)$, $k_{TE}(\xi)$, $\varphi_{En}(\xi)$ practically converge in the third approximation (here the second approximation differs form the third by 10-20%). For a mixture consisting of practically just neutral components when $x_E \leqslant 10^{-5}$, the coefficients $k_{Ti}(\xi)$ and $\varphi_{ij}(\xi)$ are negligible; consequently thermodiffusion and the correction coefficients $\varphi_{ij}(\xi)$ to the binary diffusion coefficients in Δ_{ij} cannot be taken into account here.

In the weakly ionized gas area ($x_E \simeq 10^{-3}$) the convergence of the coefficients $k_{Ti}(\xi)$ and $\varphi_{ij}(\xi)$ is poor. It turns out that for weakly ionized air, in a computation of $\lambda(\xi)$, $k_{Ti}(\xi)$, and $\varphi_{ij}(\xi)$ in a fourth approximation ($\xi = 4$) a singularity appears which is manifested by the fact that the principal determinant $\det||q_{rs}||$ in the denominators of (1.13) and (1.14) vanishes. In a practical computation of the coefficients φ_{EN}, say, this singularity is apparent as an abrupt rise in the values of the coefficients in a narrow region in the neighborhood of this point. The behavior of the coefficients φ_{EN} in different approximations ξ is displayed in the figure: Values of φ_{EN} computed in a second approximation in ξ are shown by solid lines, while values of φ_{EN} computed in a fourth approximation are shown by dashes. Disappearance of the denominator in the transport coefficients for $\xi = 4$ is apparently associated with the nature of the behavior of the interaction sections of the ionized nitrogen components since for $\xi = 4$ the appropriate formulas have a singular point for weakly ionized nitrogen, while no such singularity occurs in the case of purely weakly ionized oxygen.

The existence of singular points and the convergence of the transport coefficients in the weak ionization domain require additional investigations and execution of computations in higher approximation of ξ ($\xi > 4$). However, it should be noted that the poor convergence of the transport coefficients in the weak ionization domain has practically no effect on the properties of the mixture as a whole since all the coefficients with poor convergence in this area ($k_{Tc}(\xi)$, $k_{Tn}(\xi)$, $x_c\varphi_{cc}(\xi)$, $x_E\varphi_{En}(\xi)$) turn out to be small and yield no noticeable contribution to the transport coefficients, while the convergence of all the coefficients becomes rapid as the degree of ionization grows.

More detailed data, needed for a detailed analysis of the transport coefficients considered, are presented in reports for the Institute of Mechanics of Moscow State University [13, 24].

Remark 1. Writing the mass transport equations of the components in the form (1.11) and (1.12) is quite convenient for the solution of problems if the element diffusion fluxes J_j^* ($j = 1, \ldots, L$), and the reaction product diffusion fluxes J_i ($i = L + 1, \ldots, N$) are introduced together with the desired concentrations c_i ($i = 1, \ldots, N$) as the new desired quantities. It is then not necessary to solve (1.11) and (1.12) for the fluxes and to substitute them into (1.5) and (1.6), but it is possible to write the diffusion-kinetic system of equations (1.5), (1.6), (1.11), (1.12) in the case of a parabolic approximation, in the normal Cauchy form for the derivatives of c_i ($i = 1, \ldots, N$) and J_j^* ($j = 1, \ldots, L$), J_i ($i = L + 1, \ldots, N$) for which effective numerical methods of solution have been developed [19]. In such an approach, multicomponent diffusion coefficients or their analogs do not appear.

Remark 2. The relationship between the coefficients $\lambda(\xi)$ and $k_{Ti}(\xi)$, respectively, and the heat conduction coefficients and the multicomponent thermodiffusion coefficients introduced in [4] is discussed in detail in [17].

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HYDRODYNAMICS OF A STRATIFIED LIQUID IN THE TERMINOLOGY OF THE LAMB MOMENTUM DENSITY

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UDC 532+533

The wave motion of a stratified fluid is not separated from the vortex component in the Navier—Stokes equations. This makes the analysis of motion difficult in the nonlinear case when the wave and vortex components can reciprocally generate each other. Consequently, a description of the nonlinear dynamics of a stratified fluid in the terminology of the velocity or vorticity fields is not optimal and selection of other variables, whose evolution in time would be mutually less dependent, is desirable.

As is shown in [1, 2], a particular class of ideal stratified media motions exists which conserve their form under arbitrary levels of nonlinearity. In an incompressible fluid these are the motions whose velocity field can be expressed in terms of the density ρ and scalar functions λ, φ by the formula [2]

$$\rho \mathbf{v} = -\nabla\varphi + \lambda\nabla\rho. \tag{1}$$

In the terminology of the functions introduced, the fluid dynamics turns out to be Hamiltonian while λ, ρ are canonically conjugate variables. The wave motions that are described by such variables possess vorticity. However, the class of motions (1) is constrained, and they can be considered analogs of potential motions of a homogeneous fluid [2].

In this paper, a representation is obtained for the velocity field of an incompressible fluid, which generalizes (1) and yields a partition of the total motion into separate components. This representation results in a natural manner from the equations of motion if they are first written in the terminology of a new variable, the Lamb momentum density. The equations obtained are converted to Hamiltonian form. They can be used to search for the Lagrange

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